

Higher-Order, Mixed Finite-Element Methods for Time-Domain Electromagnetics

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Time-domain computational electromagnetic (CEM) modeling is the most cost-effective approach for electromagnetic design and analysis for problems that are electromagnetically very large or nonlinear and transient in nature. However, no stable, higher-order, conservative methods—such as those that exist for computational fluid dynamics and other disciplines—exist for time-domain CEM, largely because of the unique characteristics possessed by Maxwell's equations, which are typically solved with a set of low-order, Cartesian-grid methods. These simple, stable methods work well for rectangular geometries but require prohibitively large meshes for quantitative field predictions on electrically large problems and produce inconsistent solutions for nonorthogonal geometries. In addition, finite-volume schemes, which are often considered as an alternative, suffer from numerical instabilities, lack of conservation, and low accuracy.

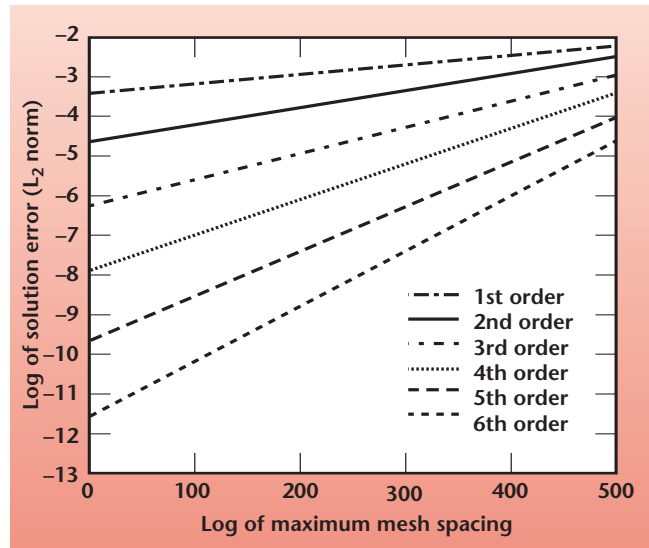
To overcome these limitations, this project is investigating higher-order, mixed finite-element methods (MFEM)—a new methodology for solving partial differential equations on three-dimensional (3-D), unstructured grids. Our goal is to develop a stable, conservative, higher-order, and accurate simulation code for Maxwell's equations and related equations of electromagnetism. Our MFEM methodology builds on LLNL's competency in computational engineering and will 1) enable engineers to perform electromagnetic simulations on unstructured grids, and 2) provide a coupled electrothermalmechanical modeling capability, both in support of the Laboratory's national security mission.

Accomplishments in FY02 include 1) making progress on the core higher-order numerical components of our simulation code, such as integration rules, basis functions, and the hexahedron, tetrahedron, and prism elements; 2) collaborating with a consulting firm to develop software components for assembling and solving, in parallel, the global linear systems; and 3) using these software components to demonstrate the efficacy of higher-order methods and perform a time-dependent simulation using higher-order time-integration methods.

To demonstrate accuracy, we used MFEM methods of various orders to solve the vector Helmholtz equation

for a resonator. The figure shows the rate of error decreasing as the mesh spacing is refined. The sixth-order method achieves, for the first time at LLNL, almost 12 digits of solution accuracy. We also studied using adaptive Runge-Kutta schemes for time-dependent problems and tested our high-order basis functions in the parallel assembly of mass and stiffness matrices for arbitrary-order, finite-element discretization.

Our work in FY03 will focus on 1) completing our prototype parallel, higher-order electromagnetic-application code; 2) further evaluating this code by comparing computed results to well-known solutions; 3) preparing and submitting journal articles about our underlying numerical methods; 4) modifying our software to improve parallel efficiency, scalability, and ease-of-use; and 5) demonstrating our technology by simulating important, longstanding issues in CEM—such as absorbing boundary conditions, mesh refinement, and material models—that existing methodologies cannot effectively simulate. 



MFEM methods of various orders to solve the vector Helmholtz equation for a resonator. The figure shows that the rate of error decreases as mesh spacing is refined, and that the sixth-order method achieves almost 12 digits of solution accuracy.